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Identification of fast charged particles in nuclear emulsions

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With 10 figures in the text

SUMMARY

Photometric grain density measurements have been made on tracks of singly charged particles not stopping in the emulsion. The method to identify particles by determining the grain density and the rate of change of grain density along the track has been studied. The method can be used for particle identification on gray tracks with $\beta \leq 0.65$. Systematic errors and errors from the statistical distribution of the grains in the tracks have been investigated. The method is less accurate than the scattering–grain density method, but it can be used as a complement to this method when unexpected results are obtained or when distorsion makes scattering measurements unreliable.

Introduction

A charged particle passing through a nuclear emulsion without stopping is normally identified by measurements of grain density and Coulomb scattering. A study of the grain density and the rate of change of grain density along the track can also give some information about the identity of a particle. For a given velocity this change is inversely proportional to the mass of the particle. Rosendorf and Yekutieli have briefly discussed this method, making a few mass determinations to show that identification is actually possible [1]. It has also been used in some cases for identification of strange particles. Nevertheless, no systematic study of the method exists. We have therefore made an investigation of its usefulness for particle identification. The grain density of about 60 tracks has been measured by means of a photometric instrument, and the method and its inherent errors have been studied.

The photometric instrument

The instrument used for the grain density measurements has been described by v. Friesen and Kristiansson 1952 [2], v. Friesen and Stigmark 1954 [3], and Kristiansson 1954 [4]. The instrument is in principle unchanged with the exception of the slit, the dimensions of which have been modified to 0.2 mm × 5 mm, which is equivalent to a slit size of $2\,\mu \times 50\,\mu$ in the objective plane. The objective is a Leitz Ks 100 ×.

A slit of these dimensions makes it impossible to measure tracks which are much inclined in relation to the plane of the emulsion [4]. The dip angle has always fulfilled the condition tg $\alpha' \leq 0.05$, where α' is the dip angle in the processed emulsion. The investigation has been made on long tracks and this dip condition does not limit the number of tracks which could be used. A shorter slit will increase the maximum dip angle allowed.

The speed of the measurements has amounted to about 10 mm/hour. The reason for the sudden rise in the velocity of the measurements (earlier 4.5 mm/hour) is the increase in the slit length, a reduction of the time constant of the recording system and furthermore the facility to adjust the very straight tracks relative to the slit [5].

The instrument has been tested according to methods earlier described by Kristiansson [5].

Selection of tracks, depth correction, the plate factor

The tracks have been found in plates exposed by the Sardinian expedition 1953, flight No. 25 II PD. The type of emulsion is Ilford G5 on glass with a thickness of 600 μ. The quality of the emulsion is high with low electron background. The plateau value of the blob density is 20 blobs/100 μ. The grain density of the tracks selected for the investigation lies between 2 and 5 times the plateau grain density. The lengths of the tracks are as a rule more than 10 mm, the average value being 22 mm. Details about the grain density and track length will be given in a later paragraph. The investigation has been restricted to tracks of singly charged particles.

The photometric measurements of mean track width (MTW) have been corrected for the depth in the emulsion. The depth correction factors were determined from proton tracks in the residual range interval 6 mm $\leq R \leq$ 10 mm according to methods described earlier [4, 5]. The same set of factors can be used for all values of MTW.

The tracks have been selected in two different plates. The development is not exactly the same in the two emulsions. A plate factor for MTW has therefore been determined, making a comparison of MTW possible between the two plates. The factor by which the MTW measurements in the first plate are multiplied in order to transform them to the degree of development in the second plate is equal to the ratio between MTW for protons at $R = 8$ mm in the first plate to the corresponding MTW in the second plate. The MTW of $R = 8$ mm has been determined as the MTW mean value in the interval 6 mm $\leq R \leq$ 10 mm. The accuracy of the factor (0.977) is high, the error being less than 1.5 %. The reasons for determining the plate factor in this way are the following: A large number of protons with residual ranges > 10 mm are easily found. The point of measurement is well defined from range measurements. The grain density is high, which reduces the error from the statistical distributions of the grains. But the grain density is not so high that the grains clog together to a very dense track. In such a case the factor may be unreliable and not useful for thinner tracks.

Method of identification from the MTW measurements

Fig. 1 is a plot of MTW versus range for a grey track. Two parameters, which can be used for the identification, are the mean value of MTW and the rate of change of MTW along the track. The first parameter is simply determined by a mean value

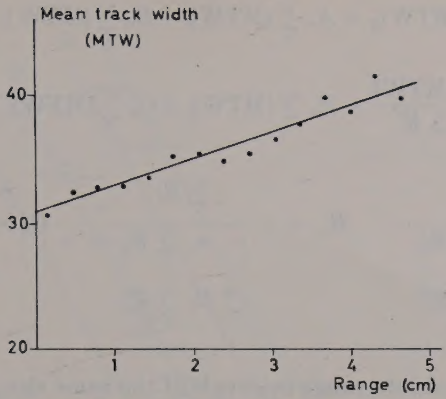

Fig. 1. Approximation of the mean track width–range relation by a straight line. The line is drawn according to the method of least squares.

calculation of the measurements of MTW. The MTW–range relation is approximated by a straight line in the diagram. The slope of this line will be proportional to the rate of change of MTW and thus inversely proportional to the mass of the particle. Accordingly this slope can be used as a second parameter in the mass determination of an unknown particle.

MTW is normalized to 100 at the residual range $R = 0$ for tracks of singly charged particles. In the same scale the plateau value is equal to about 15 units. The rate of change of MTW will be given in units of MTW/cm.

The slope has been calculated by the method of least squares. The straight line which approximates the MTW–range relation can be written

$$(\mathrm{MTW}) = (\mathrm{MTW})_0 + \frac{\Delta\,(\mathrm{MTW})}{\Delta\,R}\,R, \tag{1}$$

where $(\mathrm{MTW})_0$ and $\Delta(\mathrm{MTW})/\Delta R$ are constants. $\Delta(\mathrm{MTW})/\Delta R$ is the slope wanted for the identification. R is the range coordinate for the gray track.

In order to reduce the numerical computations, consecutive measurements are collected into groups and the mean value of MTW in each group is calculated. These mean values are used in the least squares calculations.

The least squares condition for the determination of the constants is that

$$\sum_i ((\mathrm{MTW}) - (\mathrm{MTW})_i)^2 \tag{2}$$

shall be minimum.

$(\mathrm{MTW})_1, R_1; (\mathrm{MTW})_2, R_2; \dots (\mathrm{MTW})_i, R_i; \dots (\mathrm{MTW})_n, R_n$ are the above-mentioned mean values of consecutive measurements with their associated range coordinates. $(\mathrm{MTW})_i$ is the mean value of MTW in the interval i, which has the range coordinate R_i at its centre. As a rule the grouping of the measurements of the track fulfills the condition $10 < n < 20$.

The expressions for $(\mathrm{MTW})_0$ and $\Delta(\mathrm{MTW})/\Delta R$, obtained from the least squares calculations, can be written

$$(\mathrm{MTW})_0 = A_n \sum_i (\mathrm{MTW})_i + B_n \sum_i (\mathrm{MTW})_i R_i , \tag{3}$$

$$\frac{\Delta\,(\mathrm{MTW})}{\Delta R} = B_n \sum_i (\mathrm{MTW})_i + C_n \sum_i (\mathrm{MTW})_i R_i , \tag{4}$$

where

$$A_n = \frac{\sum_i R_i^2}{\begin{vmatrix} n & \sum_i R_i \\ \sum_i R_i & \sum_i R_i^2 \end{vmatrix}}, \qquad B_n = -\frac{\sum_i R_i}{\begin{vmatrix} n & \sum_i R_i \\ \sum_i R_i & \sum_i R_i^2 \end{vmatrix}}, \qquad C_n = \frac{n}{\begin{vmatrix} n & \sum_i R_i \\ \sum_i R_i & \sum_i R_i^2 \end{vmatrix}}.$$

If the track is divided into range intervals of the same size, $R_1 \ldots R_i \ldots R_n$ can be substituted by $r \cdot 1 \ldots r \cdot i \ldots r \cdot n$. This being done, A_n, $r \cdot B_n$ and $r^2 \cdot C_n$ are functions which depend only on the number of intervals (n). They can be found directly in least squares tables [6] or are easily calculated.

The measurements

The diagram in Fig. 2 shows the results of the measurements of about 60 tracks. The particles in the large group are expected to be protons. This is confirmed by the small π-meson group. The positions of these mesons are correct if the large group contains proton measurements only. The π-mesons have been followed to the ends of their ranges and have been identified either by their decay or their interaction. It has been possible to test the position of the proton curve at the highest MTW values in the diagram by using identified protons which stop in the emulsion. If the rate of change of their MTW–range relation is calculated and plotted in the figure, it falls just on the proton curve. It is possible to calculate the direction of the curve at low MTW values if it is assumed that there is proportionality between MTW and energy loss in tracks in this energy loss interval [7]. The plateau value has been determined from tracks of electrons from μ-meson decay.

A few relativistic α-particles have also been measured. Their MTW positions are marked by arrows in the diagram. They do not show any slope significantly different from 0.

The position of the proton group relative to the π-mesons is a good proof that the identification is correct. But if there should still remain a slight suspicion that the group might be a mixture of protons, deuterons, and tritons and that two systematic errors have cancelled each other, one more method of identification is possible. Fig. 3 shows scattering measurements of about 20 particles belonging to the proton group in Fig. 2. We believe that this diagram really proves that the assumed proton group in Fig. 2 contains only one kind of particle.

The MTW–$p\beta$ relation for protons in Fig. 3 has been calculated from the proton curve in Fig. 2. This latter relation has been integrated. The integration will be discussed in a later section. It gives a MTW–range relation for protons which has been transformed to a MTW–$p\beta$ relation and plotted in Fig. 3. The agreement between the measurements and the calculated curve is excellent. This agreement is a definite proof of the correctness of our identification of the proton group.

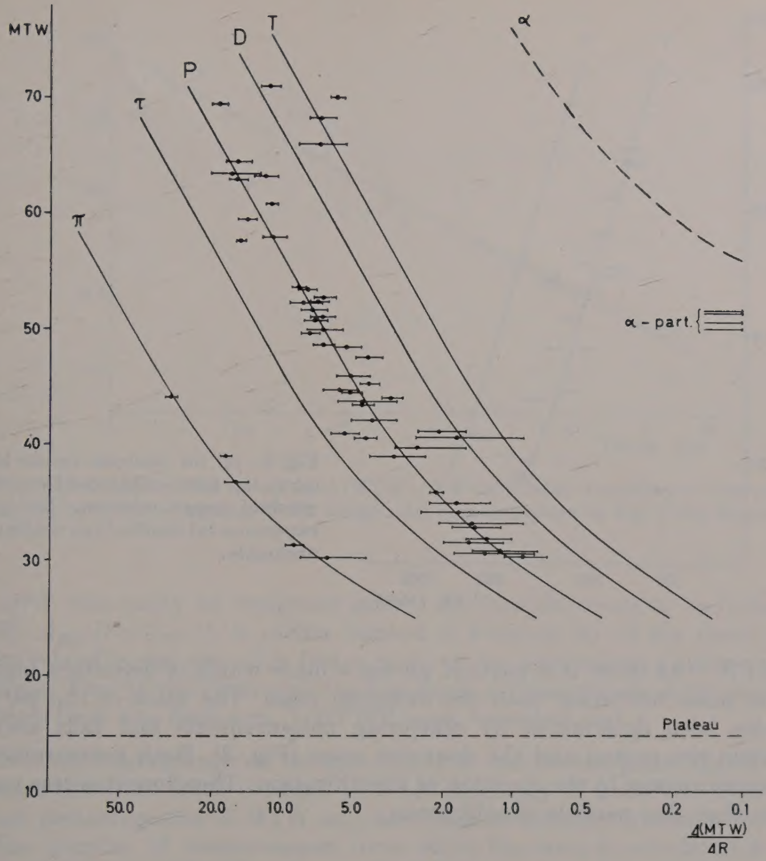

Fig. 2. The mean value of mean track width (MTW) plotted against rate of change of MTW. The particles in the large group are expected to be protons. The identity of the particles in the π-meson group is well established. The curve for the α-particles is calculated and only approximative.

The scattering measurements have been made on a microscope with a noise of 0.12 μ according to Fowler's coordinate method with a $4\bar{D}$ cut off [8]. The number of measurements on each track has been about 70, thus giving an error in $p\beta$ of about 10 %. The scattering constant has been equal to 25.0 and the cell size 200 μ.

In Fig. 2 a particle is found at a MTW of about 58 giving a mass which is very much lower than the proton mass. It has been identified by scattering–MTW measurements as a particle of protonic mass. The discrepancy can be explained as follows. The track belonged to a star very near one of the edges of the emulsion. It was directed at right angles to the edge. If there is a gradient in the development near the edges, this gradient will be added to the ordinary gradient of grain density in the tracks and it will be impossible to identify a particle according to the method described. This gradient in the development seems to be negligible, if the distance from the edge to the track is 1 cm or more.

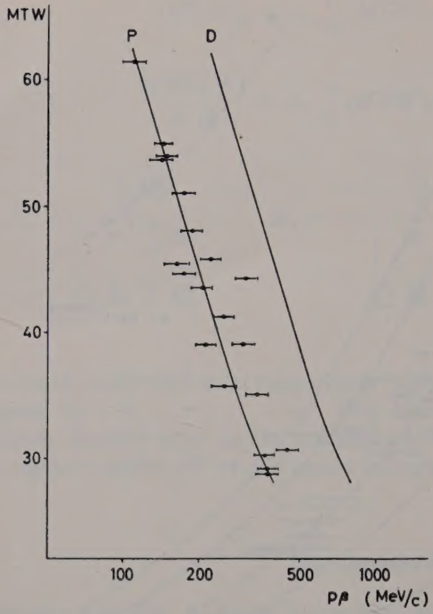

Fig. 3. $p\beta$ for protons versus MTW. The curve has been calculated from the MTW–residual range relation. The points are experimental results from scattering measurements.

At a MTW = 44 there is a particle giving a mass which is definitely higher than the proton mass but lower than the deuteron mass. The mass of the particle has furthermore been determined by scattering measurements and falls also in this case between the proton and the deuteron mass (Fig. 3). Both measurements thus give the same answer to the question of identification. Therefore it seems reasonable to assume that this particle is a hyperon.

The accuracy of the mass determinations

The error in the mass determination is chiefly determined by the accuracy with which the rate of change of MTW can be measured. This depends on the normal distribution of the MTW values, the number of measurements and also on irregularities in the emulsion.

The error (ε) in $\Delta(\text{MTW})/\Delta R$ has been calculated from a formula

$$\varepsilon = \mu \sqrt{\frac{A_{22}}{D}}, \qquad (5)$$

where

$$D = \begin{vmatrix} n & \sum_i R_i \\ \sum_i R_i & \sum_i R_i^2 \end{vmatrix},$$

$$A_{22} = n,$$

$$\mu = \sqrt{\frac{\sum_i (\text{MTW})_i^2 - (\text{MTW})_0 \sum_i (\text{MTW})_i - \frac{\Delta(\text{MTW})}{\Delta R} \sum_i (\text{MTW})_i R_i}{n-2}}.$$

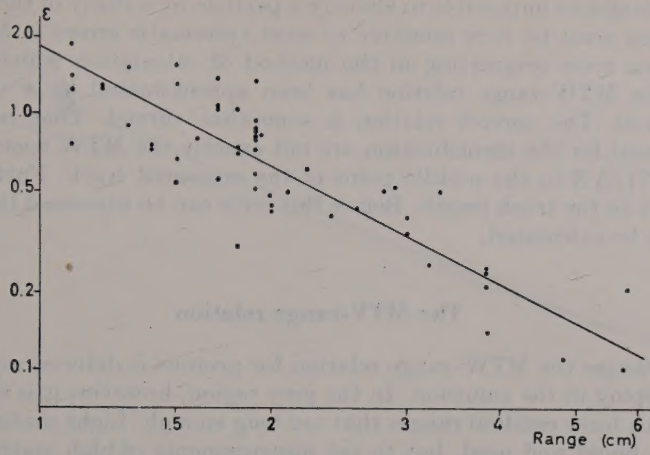

Fig. 4. The distribution of the errors in $\Delta(\mathrm{MTW})/\Delta R$ calculated according to formula (5). The errors are plotted against track length. All measurements in Fig. 2 are included.

The error can easily be computed even if the formula seems to be rather complicated [9]. $A_{22}/D = C_n$. C_n is earlier defined in formula (4). If the track is divided into intervals of equal size, this factor is given by least squares tables. μ contains three terms. Two of the sums and the constants are computed in the least squares calculation. Only the first sum $\sum_i(\mathrm{MTW})_i^2$ must be calculated.

The formula is advantageous because the error can be computed direcly from the measurements on the track studied. It is not necessary to know the distribution of individual measurements of MTW and how this distribution varies with MTW.

But the number of measurements from which the error is calculated is comparatively small ($10 < n < 20$). That means that the error is less exactly determined than if a well-determined standard deviation of the measurements was used. Accordingly ε will show a distribution around its correct value. The width of this distribution is shown in Fig. 4 where ε calculated by formula (5) for each track in Fig. 2 is plotted against track length. The standard error in ε amounts to about 25 %.

All errors in $\Delta(\mathrm{MTW})/\Delta R$ in Fig. 2 are calculated with the above-mentioned formula. About 70 % of the measurements in the proton group have a deviation from the proton curve less than one standard deviation. This number is not different from the value to be expected in a normal distribution. The conclusion drawn is that the formula is suitable for the calculation of the errors. The formula is applicable only if the MTW values in the calculations are uncorrelated and thus independent of each other. Extended irregularities in the emulsion causing systematic errors in MTW must therefore be small and unimportant in comparison to the error from the normal distribution of the measurements which chiefly depends on the distribution of the grains in the track.

The error in the mean value of MTW is small, 1–2 %, and not very important for the identification. It depends on the grain density of the track and on the number of measurements. Systematic errors from the emulsion, from the correction for depth or errors from the photometric instrument must necessarily be very small. If they were large it would be impossible to identify a particle by a study of the rate of change of MTW which must be very sensitive to most systematic errors in MTW.

There is one error originating in the method of calculation which must also be discussed. The MTW–range relation has been approximated by a straight line in the calculations. The correct relation is somewhat curved. That means that the parameters used for the identification are not exactly the MTW coordinate and the slope $\Delta(\mathrm{MTW})/\Delta R$ in the middle point of the measured track. Furthermore, they are dependent on the track length. Before this error can be discussed the MTW–range relation must be calculated.

The MTV-range relation

For short ranges the MTW–range relation for protons is determined directly from particles stopping in the emulsion. In the grey region, however, it is difficult to find particles which have residual ranges that are long enough. Light mesons of sufficient range can be found and used, but to get measurements of high statistical weight a large number of such particles is necessary. In our case it was impossible to obtain a sufficient number. Instead the relation was computed by an integration of the proton curve in Fig. 2. The function can be written

$$\frac{d\,(\mathrm{MTW})}{d\,R} = f\,(\mathrm{MTW}). \tag{6}$$

The discussion in the next section will show that $\Delta(\mathrm{MTW})/\Delta R$ can be substituted by $d(\mathrm{MTW})/dR$ without introducing serious errors. After integration we have

$$R = R_p + \int\limits_{(\mathrm{MTW})_p}^{\mathrm{MTW}} \frac{d\,(\mathrm{MTW})}{f\,(\mathrm{MTW})}. \tag{7}$$

One of the limits of integration is R_p, $(\mathrm{MTW})_p$, which is determined from protons stopping in the emulsion. ($R_p = 1$ cm, $(\mathrm{MTW})_p = 65.5$).

In the region $65.5 > \mathrm{MTW} > 42$, $f(\mathrm{MTW})$ can be approximated by a straight line in a log-line-diagram, the integration being easily performed. In the region $\mathrm{MTW} < 42$ the calculation of the range has been made by numerical integration. The result is plotted in Fig. 5.

The error in the range is defined by the quotient $\delta f(\mathrm{MTW})/f(\mathrm{MTW})$ along the track where $\delta f(\mathrm{MTW})$ is the error in the slope. From the distribution of the measurements it can be assumed that this quotient hardly exceeds 8 % at any MTW. It is expected to be positive and negative alternately. That will reduce the error in R, probably to less than 5 %. A more precise estimation of the error in R is very difficult to perform.

In diagram 5 also energy loss is plotted against residual range. The position of the energy loss scale to the right in the figure is adjusted until the two curves coincide at large ranges and low energy losses. The emulsion tables calculated by Barkas and Young [10] have been used for the energy loss scale. The diagram shows that MTW and energy loss are roughly proportional up to an energy loss of about three times minimum.

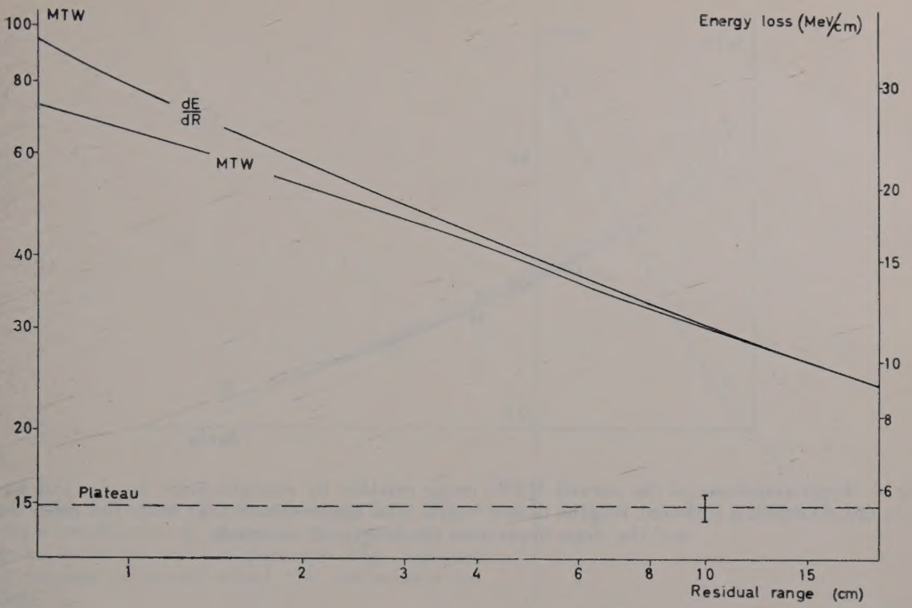

Fig. 5. MTW–residual range and energy loss–residual range for protons. The position of the energy loss scale has been adjusted until the two curves coincide at large ranges and low energy losses.

Correction for the curvature of the MTW-range relation

If diagram 5 had been plotted with linear axes the MTW–range relation would have been much more curved. In the calculations of the slope and the mean value of MTW for the identification of a particle the MTW–range relation for the track is approximated by a straight line in a linear scale. To get parameters well defined for the identification we need the MTW of the middle point of the MTW–range curve for the track and the direction of the tangent to the curve in the same point. The above-mentioned mean value and the slope of the straight line do not coincide exactly with these parameters. The differences depend on the length of the track. Fig. 6 illustrates the approximations of the curve by straight lines. A–B is the MTW–range relation in a linear diagram. For identification the slope and the MTW value at M are wanted. When tracks of different lengths are measured and approximated by straight lines L_1, L_2, and L_3, it can easily be seen from the figure how both the mean value and the slope change with the length of the track.

The corrections for curvature which must be subtracted from the mean value and the slope in order to eliminate the influence of track length are shown in Fig. 7A and B. They are calculated for protons and given as functions of MTW and with the track length L (in cm) as a parameter. Figure 8 shows some examples of how much points in diagram 2 will be moved when the corrections are applied. The points are so chosen that the largest possible corrections are shown for the different track lengths. The largest MTW mean value for each track length is obtained at the extreme low energy end of the particle.

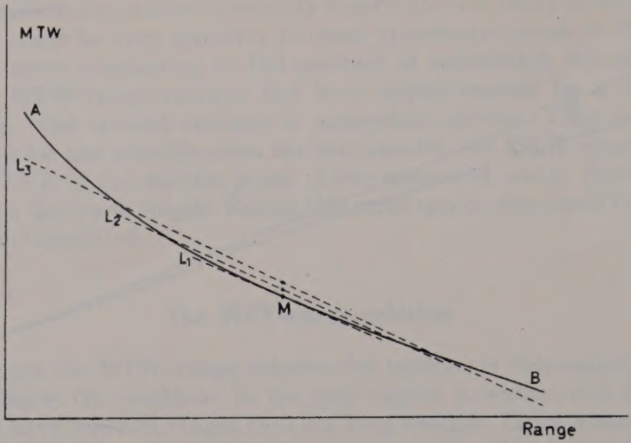

Fig. 6. Approximations of the curved MTW–range relation by straight lines. L_1, L_2, and L_3 are lines approximating different lengths of the curve. The figure shows that both the mean value and the slope depend on the length of the track.

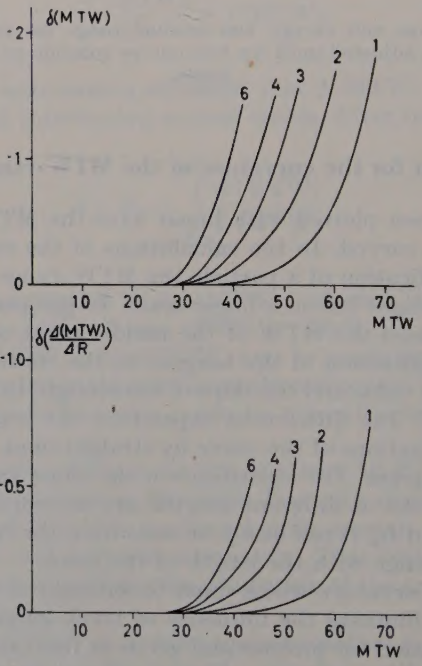

Fig. 7, A and B. Corrections which must be subtracted from the mean value of MTW and $\Delta(\mathrm{MTW})/\Delta R$ because of the straight line approximation. The corrections are calculated for protons and given as functions of the mean value of MTW. The parameter indicates the track lengths in cm. Fig. A shows the reduction of the mean value of MTW and Fig. B the corresponding correction of $\Delta(\mathrm{MTW})/\Delta R$.

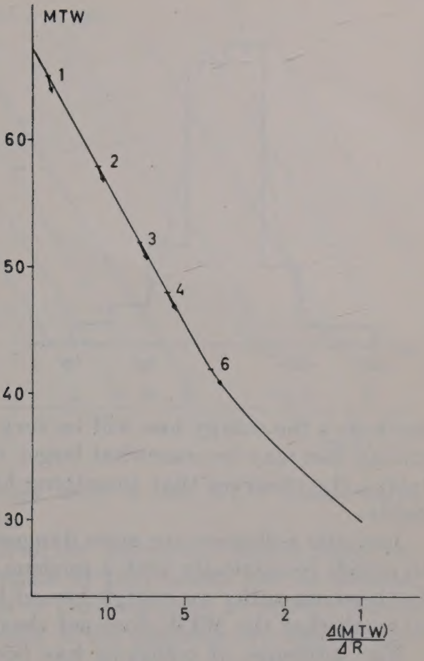

Fig. 8. The influence of the correction from Fig. 7 on the identification of particles. The arrows in the diagram show the shift of points in Fig. 2 at the track lengths indicated when the corrections are applied.

The corrections depend on the mass of the particle. They can be determined directly from Fig. 7 for particles of different masses if the range parameter L is substituted by $L'(M_p/M')$, where L' is the length of a track made by a particle with the mass M'. $\delta(\Delta(\mathrm{MTW})/\Delta R)$ has furthermore to be multiplied by M_p/M'. The corrections are always small and unimportant for the identification. If the corrections are applied, the curve in Fig. 2 will not change its position by an amount greater than the uncertainty of the curve.

It is worth noticing that if the velocity is determined for a particle having a long track by a measurement of the mean value of the grain density, and if this mean value and the velocity of the particle are referred to the middle point of the track, a serious error may be introduced if the correction is not taken into consideration.

Influence of collisions on the identification

The identification is very sensitive to an abrupt loss of energy. A very large energy loss along the track will make the change in MTW too rapid and accordingly the mass of the particle will come out too low.

Such an energy loss depends on an elastic or inelastic collision and is often accompanied by a large angle scattering. In order to avoid as many as possible of these collisions, only those tracks are used which do not show large angle scatterings of more than 5°.

Elastic collisions between the particle and the nuclei in the emulsion are not very dangerous, when the above-mentioned scattering angle is an upper limit. If the nuclei

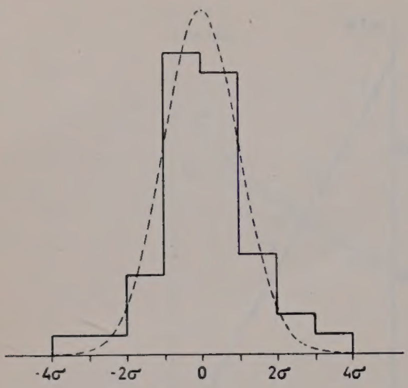

Fig. 9. The mass distribution of the particles in the proton group in Fig. 2. The deviation from the proton curve is expressed in units of the standard error of each mass determination. The dotted curve is the normal distribution.

are heavy the energy loss will be very low. If the particle collides with a proton, the energy loss may be somewhat larger but in this case the proton track is easily seen, telling the observer that something has happened to make the measurements unreliable.

Inelastic collisions are more dangerous. But the probability is small for a particle to collide inelastically with a nucleus and continue in nearly the same direction and furthermore suffer an energy loss so large that the identification will be wrong, yet so small that the MTW does not change discontinuously with an observable amount.

The influence of collisions has been tested on the proton group in Fig. 2. Fig. 9 shows the distribution of the proton masses. The deviation from the proton curve in Fig. 2 is given in units of the standard error of each mass determination. Inelastic collisions would make the mass apparently too low with a tail to the left of the mass distribution in the histogram. There is no indication of any tail against low masses which supports the conclusion that the energy loss by collisions with the nuclei in the emulsion is as a rule not very dangerous in particle identification by this method.

The particle which deviates more than 3σ from the mean has been discussed earlier. Its abnormal deviation depends on a gradient in the development near the edge of the plate.

Concluding remarks

It is obvious from Figures 2 and 9 that it is possible to identify particles by studying MTW and the rate of change of MTW. But if high accuracy is wanted, comparatively long tracks are needed.

The error (ε) in $\Delta(\mathrm{MTW})/\Delta R$ is plotted in Fig. 4 against track length. The figure shows a dependence between R and $\varepsilon(R)$

$$\varepsilon(R) = \frac{\varepsilon(R=1)}{R^{3/2}}. \qquad (8)$$

$\varepsilon(R)$ does not vary with MTW. The reason is that the standard deviation of the MTW measurements is nearly constant and independent of MTW in the whole gray region.

The influence of $\varepsilon(R)$ on the identification of different particles is shown in Fig. 10. The error to be expected in a mass determination is plotted against track length for

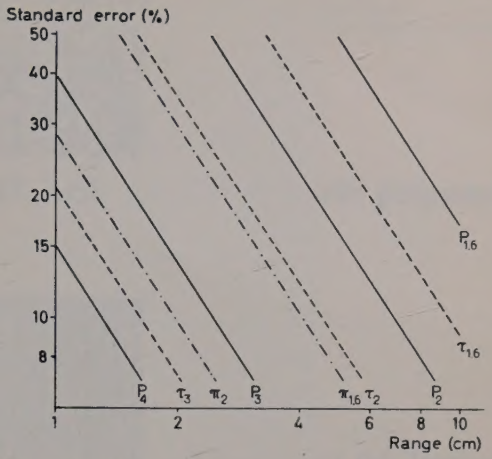

Fig. 10. Errors to be expected in mass determinations according to the method described. The percentage error is plotted against track length for protons, τ-mesons and π-mesons with the MTW mean value of the track as a parameter. The indices belonging to P, τ, and π indicate this MTW mean value in units of the MTW plateau value.

π-mesons, τ-mesons and protons. The parameter used is the MTW mean value divided by the MTW plateau value for the plate.

The energy loss region suitable for the method lies between 2 and 5 times the minimum energy loss; thus nearly the whole region of gray tracks can be studied. If very long tracks are available, identifications of particles with an energy loss as low as 1.75 times minimum are possible, especially if the particles are lighter than protons.

The accuracy of mass determinations according to the method is somewhat lower than the accuracy in measurements of scattering combined with MTW. The method seems to be important as a complementary method. A strange result obtained in a mass determination by scattering can be tested fairly accurately. It can furthermore be used in plates with severely distorted tracks where scattering measurements are unreliable.

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